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Mathematical modeling of the intervertebral disc nutritional transport

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INTRODUCTION: As there is no blood supply to the main body of the human intervertebral disc (IVD), the disc cells rely on diffusion and convection for the transport of nutrients and wastes. Difficulties associated with existing experimental techniques render mathematical modeling an attractive method for investigating the mechanical environment of the IVD and predicting nutrient transport. Commercial software package such as ABAQUS have been used to simulate nutrient transport under steady state [3] and dynamic [1] conditions. In this paper, we generalize the theoretical model [2] to include nutritional transport mechanism.

METHODS: We use the theory of mixture to develop a two phase mathematical model incorporating strain related osmotic swelling pressure and permeability, together with nutrients transport with convection and diffusion mechanism. The governing equations are:

Balance of mass

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \frac{\partial U}{\partial t}) = 0 \qquad \text{Fluid phase}$$

$$\frac{\partial (1-\phi)}{\partial t} + \nabla \cdot ((1-\phi)\frac{\partial u}{\partial t}) = 0 \qquad \text{Solid phase}$$

Balance of momentum

$$\nabla \cdot (\phi \sigma_f) + F_f = 0 \qquad \text{Fluid phase}$$

$$\nabla \cdot ((1-\phi)\sigma_s) + F_s = 0 \qquad \text{Solid phase}$$

Constitutive laws

$$F_f = -F_s, F_f = P_f \nabla \cdot \phi + D1$$

$P_f \nabla \cdot \phi$ is interfacial force, $D1$ is a drag term:

Nutritional transport equation

$$\frac{\partial C}{\partial t} + \nabla \cdot \{C(\frac{\partial U}{\partial t} - \frac{\partial u}{\partial t})\} = \nabla \cdot (D \nabla C) - R(C)$$

Where ϕ, U, u, C denote the porosity, fluid displacement, solid displacement and nutrient concentration.

RESULTS: Numerical simulation has been employed to reduced one-d case. A sample disc was under 0.8MPa loading by 20 minutes and 40 minutes recovery. The disc height change and nutrient transport have been calculated.

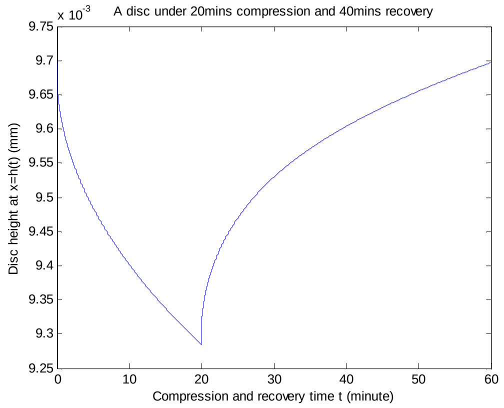

Fig. 1 The disc height change curve

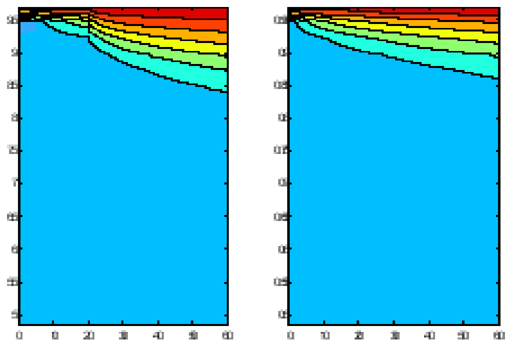

Fig. 2 Comparison of penetration between convection coupled nutrient diffusion and diffusion without fluid flow (D=60 $\mu m^2/s$).

DISCUSSION & CONCLUSIONS: Our model agrees with the calculation in [1] that convection enhances the transport for large molecules significantly but has little effect for small molecules transport.

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